

ASPECTS OF NUCLEON COMPTON SCATTERING

V. Bernard¹, N. Kaiser², Ulf-G. Meißner¹, A. Schmidt²

¹*Centre de Recherches Nucléaires et Université Louis Pasteur de Strasbourg
Physique Théorique, BP 20Cr, 67037 Strasbourg Cedex 2, France*

²*Physik Department T30, Technische Universität München
James Franck Straße, D-85747 Garching, Germany*

ABSTRACT

We consider the spin-averaged nucleon forward Compton scattering amplitude in heavy baryon chiral perturbation theory including all terms to order $\mathcal{O}(q^4)$. The chiral prediction for the spin-averaged forward Compton scattering amplitude is in good agreement with the data for photon energies $\omega \leq 110$ MeV. We also evaluate the nucleon electric and magnetic Compton polarizabilities to this order and discuss the uncertainties of the various counter terms entering the chiral expansion of these quantities.

I. INTRODUCTION AND SUMMARY

Compton scattering off the nucleon at low energies offers important information about the structure of these particles in the non-perturbative regime of QCD. The spin-averaged forward scattering amplitude for real photons in the nucleon rest frame can be expanded as a power series in the photon energy ω ,

$$\begin{aligned} T(\omega) &= f_1(\omega^2) \vec{\epsilon}_f^* \cdot \vec{\epsilon}_i \\ f_1(\omega^2) &= a_0 + a_1 \omega^2 + a_2 \omega^4 + \dots \end{aligned} \tag{1}$$

where $\vec{\epsilon}_{i,f}$ are the polarization vectors of the initial and final photon, respectively, and due to crossing symmetry only even powers of ω occur. The Taylor coefficients a_i encode the information about the nucleon structure. The first term in eq.(1), $a_0 = -e^2 Z^2 / 4\pi m$, dominates as the photon energy approaches zero, it is only sensitive to the charge Z and the mass m of the particle the photon scatters off (the Thomson limit). The term quadratic in the energy is equal to the sum of the so-called electric ($\bar{\alpha}$) and magnetic ($\bar{\beta}$) Compton polarizabilities, $a_1 = \bar{\alpha} + \bar{\beta}$. Corrections of higher order in ω start out with the term proportional to a_2 .

In ref.[1], we calculated the electric and magnetic polarizabilities of the proton and the neutron in heavy baryon chiral perturbation theory (CHPT) to order q^4 (with q denoting a small external momentum or meson mass). The chiral prediction for the electric polarizabilities $\bar{\alpha}_p$ and $\bar{\alpha}_n$ are in good agreement with the available experimental data. Furthermore, we found a non-analytic term of the type $\ln M_\pi$ (with M_π the pion mass) which cancels parts of the large positive $\Delta(1232)$ contribution to the magnetic polarizability $\bar{\beta}_p$ (in case of the neutron, the coefficient of the $\ln M_\pi$ term is somewhat too small). This is a novel effect which allows to understand the relative smallness of the nucleons' magnetic polarizabilities. The aim of the present paper is twofold. First, we wish to consider the spin-averaged forward Compton amplitude as a function of the photon energy to investigate the importance of the ω^4 , ω^6 , \dots terms and to confront it with the available experimental data. Second, as already stressed in ref.[1], we have to present a detailed analysis of the uncertainties entering the theoretical predictions for the polarizabilities at order q^4 .

Heavy baryon CHPT is a systematic expansion of low-energy QCD Green functions in small external momenta and quark masses. It was previously used in the calculation of the baryon mass spectrum [2], axial currents in flavor SU(3) [3], nuclear forces [4] and many other observables (for a review, see ref.[5]). A systematic analysis of QCD Green

functions for nucleons in flavor SU(2) can be found in [6]. We will heavily borrow from that paper. We will work in the one loop approximation [6] which still allows to include all terms of chiral order q^4 . The two loop contributions start at $\mathcal{O}(q^5)$. The nucleon electromagnetic polarizabilities belong to the rare class of observables which to leading order in the chiral expansion are given as pure loop effects [7] but at order q^4 counter terms enter. Their coefficients have already been determined in [1] by either direct fits to existing data for other reactions or by making use of the resonance saturation hypothesis [8,9]. This will be discussed in more detail below.

The pertinent results of this investigation can be summarized as follows:

- (i) The spin-averaged forward Compton amplitude for the proton is in agreement with the data up to photon energies $\omega \simeq M_\pi$. It is dominated by the quadratic contribution, i.e. to order q^4 in the chiral expansion the terms of order ω^4 (and higher) are small. Similar trends are found for the neutron with the exception of a too strong curvature at the origin.
- (ii) The electromagnetic polarizabilities are in good agreement with the data (with the exception of $\bar{\beta}_n$), see eqs.(28) and (29). We have discussed the theoretical uncertainties to order q^4 and found that the electric and magnetic polarizabilities can be predicted with an accuracy of $\pm 2 \cdot 10^{-4} \text{ fm}^3$ and $\pm 4 \cdot 10^{-4} \text{ fm}^3$, respectively. An improved theoretical prediction is only possible if one can pin down the coefficients of some counter terms more accurately.

The paper is organized as follows. In section 2, we briefly discuss the effective meson-baryon lagrangian underlying our calculation. In section 3 we introduce the spin-averaged forward Compton tensor which contains the information about the amplitude $f_1(\omega^2)$ as well as the electromagnetic polarizabilities $\bar{\alpha}$ and $\bar{\beta}$. Section 3 contains the results for the spin-averaged forward Compton scattering amplitude. The theoretical predictions for the electromagnetic polarizabilities are then given in section 4.

2. EFFECTIVE LAGRANGIAN

The basic tool to investigate the low-energy behaviour of QCD Green functions is an effective Lagrangian formulated in terms of the asymptotically observed fields, the Goldstone bosons of spontaneous chiral symmetry breaking (pions) and the matter fields (the baryons). The construction principles for building a string of terms with increasing number of derivatives are chiral symmetry, lorentz invariance and the pertinent discrete symmetries of the strong interactions as well as the systematic chiral power counting

scheme (see [5] and references therein). For our purpose, it suffices to stress that we work in the one loop approximation including all terms of order q^4 . As stressed in [3], n loop graphs start to contribute at order q^{2n+1} if one inserts vertices from the lowest order effective Lagrangian $\mathcal{L}_{\pi N}^{(1)}$. Notice that we consider the two flavor case and work in the isospin limit $m_u = m_d$. The pion fields are collected in the SU(2) matrix

$$U(x) = \exp[i\vec{\tau} \cdot \vec{\pi}(x)/F] = u^2(x) \quad (2)$$

with F the pion decay constant in the chiral limit. The nucleons are considered as very heavy [2,3,4,5,6]. This allows for a projection onto velocity eigenstates and one therefore can eliminate the troublesome nucleon mass term (of the Dirac lagrangian for nucleons) thereby generating a string of vertices of increasing power in $1/m$, with m the nucleon mass (in the chiral limit).¹⁾ The effective Lagrangian to order $\mathcal{O}(q^4)$, where q denotes a genuine small momentum or a meson (quark) mass, reads (we only exhibit those terms which are actually needed in the calculation of forward Compton amplitudes) [1]

$$\begin{aligned} \mathcal{L}_{\pi N} &= \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(4)} \\ \mathcal{L}_{\pi N}^{(1)} &= \bar{H}(iv \cdot D + g_A S \cdot u)H \\ \mathcal{L}_{\pi N}^{(2)} &= \bar{H} \left\{ -\frac{1}{2m} D \cdot D + \frac{1}{2m} (v \cdot D)^2 + c_1 \text{Tr} \chi_+ + \left(c_2 - \frac{g_A^2}{8m}\right) v \cdot u v \cdot u \right. \\ &\quad \left. + c_3 u \cdot u - \frac{ig_A}{2m} \{S \cdot D, v \cdot u\} - \frac{ie}{4m} [S^\mu, S^\nu] \left((1 + \kappa_v) f_{\mu\nu}^+ + \frac{\kappa_s - \kappa_v}{2} \text{Tr} f_{\mu\nu}^+ \right) \right\} H \\ \mathcal{L}_{\pi N}^{(4)} &= \frac{\pi}{4} (\delta\bar{\beta}_p - \delta\bar{\beta}_n) \bar{H} f_{\mu\nu}^+ f_+^{\mu\nu} H + \frac{\pi}{4} \delta\bar{\beta}_n \bar{H} H \text{Tr} f_{\mu\nu}^+ f_+^{\mu\nu} \\ &\quad + \frac{\pi}{2} (\delta\bar{\alpha}_n + \delta\bar{\beta}_n - \delta\bar{\alpha}_p - \delta\bar{\beta}_p) \bar{H} f_{\mu\nu}^+ f_+^{\lambda\nu} H v^\mu v_\lambda \\ &\quad - \frac{\pi}{2} (\delta\bar{\alpha}_n + \delta\bar{\beta}_n) \bar{H} H \text{Tr} (f_{\mu\nu}^+ f_+^{\lambda\nu}) v^\mu v_\lambda \end{aligned} \quad (3)$$

with

$$\begin{aligned} u_\mu &= iu^\dagger \nabla_\mu U u^\dagger \\ f_{\mu\nu}^+ &= (\partial_\mu A_\nu - \partial_\nu A_\mu)(uZu^\dagger + u^\dagger Zu) \end{aligned} \quad (4)$$

where H denotes the heavy nucleon field of charge $Z = (1 + \tau_3)/2$ and anomalous magnetic moment $\kappa = (\kappa_s + \tau_3 \kappa_v)/2$, v_μ the four-velocity of H , S_μ the covariant spin-operator subject to the constraint $v \cdot S = 0$, ∇_μ the covariant derivative acting on

¹⁾ Later on, we will identify m with the physical nucleon mass. This is a consistent procedure to the order we are working.

the pions, $D_\mu = \partial_\mu + \Gamma_\mu$ the chiral covariant derivative for nucleons and we adhere to the notations of ref.[6]. The superscripts (1,2,4) denote the chiral power. The lowest order effective Lagrangian is of order $\mathcal{O}(q)$. The one loop contribution is suppressed with respect to the tree level by q^2 thus contributing at $\mathcal{O}(q^3)$. In addition, there are one loop diagrams with exactly one insertion from $\mathcal{L}_{\pi N}^{(2)}$. These are of order q^4 . Finally, there are contact terms of order q^2 and q^4 with coefficients not fixed by chiral symmetry. For the case at hand $\mathcal{L}_{\pi N}^{(3)}$ does not have to be considered explicitly. To order q^4 its vertices enter only tree diagrams and it therefore contributes to the polynomial part of the amplitude. The most general polynomial piece at order q^4 is, however, already given by the contact vertices of $\mathcal{L}_{\pi N}^{(4)}$. Notice that some coefficients in $\mathcal{L}_{\pi N}^{(2)}$ related to the $\gamma\gamma NN$ and $\gamma\pi NN$ vertices are fixed from the relativistic theory by the low-energy theorems for Compton scattering ($a_0 = -e^2 Z^2/4\pi m$) and neutral pion photoproduction, respectively. This is discussed in some detail in ref.[6]. The unknown coefficients we have to determine are c_1 , c_2 and c_3 characterizing a higher derivative $\pi\pi NN$ vertex as well as the four low-energy constants $\delta\bar{\alpha}_p$, $\delta\bar{\alpha}_n$, $\delta\bar{\beta}_p$ and $\delta\bar{\beta}_n$ from $\mathcal{L}_{\pi N}^{(4)}$. We have not exhibited the standard meson Lagrangian $\mathcal{L}_{\pi\pi}^{(2)}$,

To calculate all terms up-to-and-including order q^4 , we have to evaluate all one loop graphs with insertions from $\mathcal{L}_{\pi N}^{(1)}$ and those with exactly one insertion from $\mathcal{L}_{\pi N}^{(2)}$. While the former scale as q^3 , the latter constitute the new contributions of order q^4 . Furthermore, there are the tree diagrams related to $\mathcal{L}_{\pi N}^{(4)}$ which are also new. It is worth to stress that in the one loop diagrams involving $\mathcal{L}_{\pi N}^{(2)}$ the anomalous magnetic moment of the nucleon appears, since the photon-nucleon vertex is expanded in the external momentum. We will come back to this point later on. Finally, it is mandatory to expand the leading order $\mathcal{O}(q)$ effective vertices as well as the nucleon propagator to include all relativistic corrections of order $1/m$. The resulting Feynman rules for the pertinent vertices and propagators are shown in fig.1. Note that we have left out all vertices which give zero contribution either due to spin-averaging or due to isospin algebra.

In eq.(3) we have expressed the effective lagrangian in terms of the physical parameters m , κ_s , κ_v and so on. For the loop diagrams this is legitimate to the order we are working. Concerning tree diagrams which are responsible for the Thomson amplitude $a_0 = -e^2 Z^2/4\pi m$, the term proportional to c_1 will shift the nucleon mass in the chiral limit by $-4c_1 M_\pi^2$ to the physical nucleon mass at this order.

3. SPIN-AVERAGED FORWARD COMPTON TENSOR

The object one has to study in order to determine the Compton amplitude $f_1(\omega^2)$ and the electromagnetic polarizabilities is the spin-averaged Compton tensor in forward direction $\Theta_{\mu\nu}$,

$$\begin{aligned}\Theta_{\mu\nu} &= \frac{e^2}{4} \text{Tr} [(1 + \not{v}) T_{\mu\nu}(v, k)] \\ &= A(\omega) g_{\mu\nu} + B(\omega) k_\mu k_\nu + C(\omega) (k_\mu v_\nu + v_\mu k_\nu) + D(\omega) v_\mu v_\nu\end{aligned}\tag{5}$$

where v and k denote the nucleon four-velocity ($v^2 = 1$) and the photon momentum ($k^2 = 0$) and $\omega = v \cdot k$. $T_{\mu\nu}(v, k)$ is the Fourier-transformed nucleon matrix element of two time-ordered electromagnetic currents,

$$T_{\mu\nu}(v, k) = \int d^4x e^{ik \cdot x} \langle N(v) | T J_\mu^{em}(x) J_\nu^{em}(0) | N(v) \rangle\tag{6}$$

Gauge invariance $\Theta_{\mu\nu} k^\nu = 0$ implies $C(\omega) = -A(\omega)/\omega$ and $D(\omega) = 0$ and therefore all information is contained in the two crossing even functions $A(\omega) = A(-\omega)$ and $B(\omega) = B(-\omega)$. This fact allows us to choose the "Coulomb gauge" $\epsilon \cdot v = 0$ for the polarization vector ϵ of the photon when calculating the auxiliary quantity $\epsilon^\mu \Theta_{\mu\nu} \epsilon^\nu = A(\omega) \epsilon^2 + B(\omega) (\epsilon \cdot k)^2$. The gauge $\epsilon \cdot v = 0$ is very convenient since it leads to a drastic simplification in our loop calculation. In this gauge the leading order photon nucleon vertex vanishes and therefore many diagrams. The function $A(\omega)$ is directly proportional to the spin-averaged forward Compton amplitude $f_1(\omega^2)$ introduced in eq.(1), namely,

$$A(\omega) = -4\pi f_1(\omega^2)\tag{7}$$

The second function $B(\omega)$ is not a physical amplitude, it only serves to calculate the magnetic polarizability.

The electric and magnetic Compton polarizabilities are defined as follows:

$$\bar{\alpha} + \bar{\beta} = -\frac{A''(0)}{8\pi}, \quad \bar{\beta} = -\frac{B(0)}{4\pi}\tag{8}$$

where the prime denotes differentiation with respect to ω .

Our task is to calculate all contributions to $A(\omega)$ and $B(\omega)$ coming from CHPT at order q^4 . These are all one loop contributions with vertices from $\mathcal{L}_{\pi N}^{(1)}$ and those with exactly one vertex from $\mathcal{L}_{\pi N}^{(2)}$ as well as the most general polynomial contribution at this order. The latter has the form $A(\omega)^{pol} = e^2 Z^2/m - 4\pi(\delta\bar{\alpha} + \delta\bar{\beta})\omega^2$ and

$B(\omega)^{pol} = -4\pi\delta\bar{\beta}$ where the constant term in $A(\omega)^{pol}$ is fixed by the low energy theorem for Compton scattering. Later we will determine the polynomial coefficients $\delta\bar{\alpha}$, $\delta\bar{\beta}$ by identifying them with the resonance contributions to the nucleon electromagnetic polarizabilities. In making this identification we have excluded any contributions from the nucleon pole graphs to the polarizabilities. This prescription is in accordance with the analysis of the Compton scattering experiments where terms coming from the anomalous magnetic moment (nucleon pole graph) are explicitly separated from the ones of the polarizabilities by the use of the Powell cross section formula [15].

4. FORWARD COMPTON AMPLITUDE

We wish to discuss the spin-averaged forward Compton amplitude $A(\omega) = -4\pi f_1(\omega)$ for real photons ($k^2 = 0$) scattering off protons or neutrons. Making use of a once-subtracted dispersion relation and the optical theorem allows to express $A(\omega)$ in terms of the total photo-nucleon absorption cross section $\sigma_{\text{tot}}(\omega)$ as

$$\begin{aligned}\text{Re } A(\omega) &= \frac{e^2 Z^2}{m} - \frac{2\omega^2}{\pi} \mathcal{P} \int_{\omega_0}^{\infty} d\omega' \frac{\sigma_{\text{tot}}(\omega')}{\omega'^2 - \omega^2} \\ \text{Im } A(\omega) &= -\omega \sigma_{\text{tot}}(\omega)\end{aligned}\tag{9}$$

for the proton and the neutron. The threshold energy for single pion photoproduction is given by

$$\omega_0 = M_\pi(1 + M_\pi/2m) .\tag{10}$$

Working in the Coulomb gauge $\epsilon \cdot v = 0$, one has to calculate 9 irreducible one loop graphs with insertions from $\mathcal{L}_{\pi N}^{(1)}$ and 67 one loop diagrams with one insertion from $\mathcal{L}_{\pi N}^{(2)}$. Furthermore there are the contact term graphs stemming from $\mathcal{L}_{\pi N}^{(4)}$, which give rise to the most general polynomial contribution at order q^4 to $A(\omega)$ and $B(\omega)$, namely, $A(\omega)^{pol} = e^2 Z^2/m - 4\pi(\delta\bar{\alpha} + \delta\bar{\beta})\omega^2$, $B(\omega)^{pol} = -4\pi\delta\bar{\beta}$.

Due to the simple structure of the loop integrals in the heavy mass formulation, one can give $A_{p,n}(\omega)$ in closed analytic form. For the proton it reads

$$\begin{aligned}A_p(\omega) &= \frac{e^2}{m} - 4\pi(\bar{\alpha} + \bar{\beta})_p \omega^2 \\ &+ \frac{e^2 g_A^2 M_\pi}{8\pi F_\pi^2} \left\{ -\frac{3}{2} - \frac{1}{z^2} + \left(1 + \frac{1}{z^2}\right) \sqrt{1 - z^2} + \frac{1}{z} \arcsin z + \frac{11}{24} z^2 \right\} \\ &+ \frac{e^2 g_A^2 M_\pi^2}{8\pi^2 m F_\pi^2} \left\{ -\frac{5}{2} + \frac{z^2}{2} (3\kappa_s + 11) + \left[\frac{2}{z} - (6 + \kappa_s)z + \frac{z}{1 - z^2} \right] \sqrt{1 - z^2} \arcsin z \right. \\ &\quad \left. + \frac{1}{2} \left(\frac{1}{z^2} - \kappa_s \right) \arcsin^2 z \right\}\end{aligned}\tag{11}$$

with $z = \omega/M_\pi$. Similarly, one has for the neutron

$$\begin{aligned}
A_n(\omega) = & -4\pi(\bar{\alpha} + \bar{\beta})_n \omega^2 + \frac{e^2 g_A^2 M_\pi}{8\pi F_\pi^2} \left\{ -\frac{3}{2} - \frac{1}{z^2} + \left(1 + \frac{1}{z^2}\right) \sqrt{1-z^2} + \frac{1}{z} \arcsin z + \frac{11}{24} z^2 \right\} \\
& + \frac{e^2 g_A^2 M_\pi^2}{8\pi^2 m F_\pi^2} \left\{ \frac{1}{2} + \frac{z^2}{6} (7 - 9\kappa_s) + \left[(\kappa_s - 2)z + \frac{z}{1-z^2} \right] \sqrt{1-z^2} \arcsin z \right. \\
& \left. + \frac{1}{2} \left(\kappa_s - \frac{1}{z^2} \right) \arcsin^2 z \right\}
\end{aligned} \tag{12}$$

Notice that the expressions for $A_{p,n}(\omega)$ diverge at $\omega = M_\pi$. This is an artefact of the heavy mass expansion. The realistic branch point coincides with the opening of the one-pion channel as given above. To cure this, let us introduce the variable

$$\zeta = \frac{z}{1 + M_\pi/2m} = \frac{\omega}{M_\pi(1 + M_\pi/2m)} = \frac{\omega}{\omega_0}. \tag{13}$$

If one now rewrites $A_{p,n}(\omega)$ in terms of ζ , the branch point sits at its proper location and $A_{p,n}(\zeta = 1)$ is finite. We have

$$\begin{aligned}
A_{p,n}(\omega) = & \frac{e^2}{2m} (1 \pm 1) - 4\pi(\bar{\alpha} + \bar{\beta})_{p,n} \omega^2 \\
& + \frac{e^2 g_A^2 M_\pi}{8\pi F_\pi^2} \left\{ -\frac{3}{2} - \frac{1}{\zeta^2} + \left(1 + \frac{1}{\zeta^2}\right) \sqrt{1-\zeta^2} + \frac{1}{\zeta} \arcsin \zeta + \frac{11}{24} \zeta^2 \right\} \\
& + \frac{e^2 g_A^2 M_\pi^2}{8\pi^2 m F_\pi^2} \left\{ -1 + \frac{10}{3} \zeta^2 + \left[\frac{1}{\zeta} - 4\zeta + \frac{\zeta}{1-\zeta^2} \right] \sqrt{1-\zeta^2} \arcsin \zeta \right. \\
& + \pi \left[\frac{1}{\zeta^2} - \frac{1}{2\zeta} \arcsin \zeta + \frac{11}{24} \zeta^2 - \frac{(1-\zeta^2)^2 + 1}{2\zeta^2 \sqrt{1-\zeta^2}} \right] \pm \left[-\frac{3}{2} + \zeta^2 \left(\frac{3}{2} \kappa_s + \frac{13}{6} \right) \right. \\
& \left. \left. + \left(\frac{1}{\zeta} - (2 + \kappa_s) \zeta \right) \sqrt{1-\zeta^2} \arcsin \zeta + \frac{1}{2} \left(\frac{1}{\zeta^2} - \kappa_s \right) \arcsin^2 \zeta \right] \right\}
\end{aligned} \tag{14}$$

where the '+'/'-' sign refers to the proton/neutron, respectively. The proper analytic continuation above the branch point $\zeta = 1$ is obtained through the substitutions $\sqrt{1-\zeta^2} = -i\sqrt{\zeta^2-1}$ and $\arcsin \zeta = \pi/2 + i \ln(\zeta + \sqrt{\zeta^2-1})$. The expressions given in eq.(14) differ from the ones in eqs.(11,12) only by terms of order q^5 (and higher) and are thus equivalent to the order we are working. We should stress that in the relativistic formulation of baryon CHPT such problems concerning the branch point do not occur [10,11]. In the heavy mass formulation we encountered this problem since the branch point ω_0 itself has an expansion in $1/m$ and is thus different in CHPT at order q^3 and q^4 .

We now present our numerical results. We always use the Goldberger-Treiman relation to express g_A/F_π as $g_{\pi N}/m$, with $g_{\pi N} = 13.40$ the strong pion-nucleon coupling constant. We also use $\kappa_s = -0.12$, $m = 938.27$ MeV and $e^2/4\pi = 1/137.036$. In fig.2a, we show the real part of $A_p(\omega)$ normalized to the Thomson limit, $e^2/m = 3.02 \mu\text{b GeV}^{-1}$, for the z and ζ expansions in comparison to the data [12] for the central value of $(\bar{\alpha} + \bar{\beta})_p = 14.0 \cdot 10^{-4} \text{ fm}^3$ of ref.[1]. Up to $\omega \simeq 100$ MeV, the agreement of the prediction with the data is good. The corrections of order ω^4 (and higher) are fairly small as shown in fig.2b. One also recognizes the unphysical behaviour of the z -expansion around $\omega = M_\pi$. Similar statements hold for the neutron exhibited in fig.3 using the value $(\bar{\alpha} + \bar{\beta})_n = 21.2 \cdot 10^{-4} \text{ fm}^3$ of ref.[1]. Using the deuteron data of Armstrong et al.[13], one finds at $\omega = 100$ MeV, $A_n/A_p = -0.35$ compared to the theoretical prediction of -0.47 . The difference is mostly due to the too large sum of the electric and magnetic polarizabilities of the neutron.

For the proton, we have also calculated the real and imaginary parts of $A(\omega)$ for $\zeta > 1$. The imaginary part starts out negative as it should but becomes positive at $\omega \simeq 180$ MeV. This is due to the truncation of the chiral expansion and can only be overcome by a more accurate higher order calculation. Consequently, the real part (normalized to the Thomson limit) stays rather flat after the branch point as shown in fig.4.

5. ELECTROMAGNETIC POLARIZABILITIES

In [1], we derived the following formulae for the electric and magnetic polarizabilities of the proton and the neutron ($i = p, n$)

$$\begin{aligned}\bar{\alpha}_i &= \frac{5Cg_A^2}{4M_\pi} + \frac{C}{\pi} \left[\left(\frac{x_i g_A^2}{m} - c_2 \right) \ln \frac{M_\pi}{\lambda} + \frac{1}{4} \left(\frac{y_i g_A^2}{2m} - 6c_2 + c^+ \right) \right] + \delta\bar{\alpha}_i^r(\lambda), \\ \bar{\beta}_i &= \frac{Cg_A^2}{8M_\pi} + \frac{C}{\pi} \left[\left(\frac{3x'_i g_A^2}{m} - c_2 \right) \ln \frac{M_\pi}{\lambda} + \frac{1}{4} \left(\frac{y'_i g_A^2}{m} + 2c_2 - c^+ \right) \right] + \delta\bar{\beta}_i^r(\lambda).\end{aligned}\tag{15}$$

with

$$\begin{aligned}C &= \frac{e^2}{96\pi^2 F_\pi^2} = 4.36 \cdot 10^{-4} \text{ fm}^2, \\ x_p &= 9, \quad x_n = 3, \quad y_p = 71, \quad y_n = 39, \\ x'_p &= 3 + \kappa_s, \quad x'_n = 1 - \kappa_s, \quad y'_p = \frac{37}{2} + 6\kappa_s, \quad y'_n = \frac{13}{2} - 6\kappa_s, \\ c^+ &= -8c_1 + 4c_2 + 4c_3 - \frac{g_A^2}{2m}.\end{aligned}\tag{16}$$

Here, λ is the scale introduced in dimensional regularization. The physical $\bar{\alpha}_i$ and $\bar{\beta}_i$ are of course independent of this scale since the renormalized counter terms $\delta\bar{\alpha}_i^r(\lambda)$ and $\delta\bar{\beta}_i^r(\lambda)$ cancel the logarithmic λ -dependence of the loop contribution. The corresponding renormalization prescription reads:

$$\delta\bar{\alpha}_i = \frac{e^2 L}{6\pi F_\pi^2} \left(c_2 - \frac{x_i g_A^2}{m} \right) + \delta\bar{\alpha}_i^r(\lambda), \quad \delta\bar{\beta}_i = \frac{e^2 L}{6\pi F_\pi^2} \left(c_2 - \frac{3x'_i g_A^2}{m} \right) + \delta\bar{\beta}_i^r(\lambda) \quad (17)$$

with

$$L = \frac{\lambda^{d-4}}{16\pi^2} \left[\frac{1}{d-4} + \frac{1}{2}(\gamma_E - 1 - \ln 4\pi) \right] \quad (18)$$

The first term on the r.h.s. of eq.(15) is, of course, the result at order q^3 [6,7,10]. The results shown in eqs.(15) have the following structure. Besides the leading $1/M_\pi$ term [6,7,10], $\mathcal{O}(q^4)$ contributions from the loops have a $\ln M_\pi$ and a constant piece $\sim M_\pi^0$. As a check one can recover the coefficient of the $\ln M_\pi$ term from the relativistic calculation [1,2] if one sets the new low energy constants c_i and $\kappa_{s,v} = 0$. In that case only the $1/m$ corrections of the relativistic Dirac formulation are treated and one necessarily reproduces the corresponding non-analytic (logarithmic) term of this approach. The term proportional to $c_2 \ln M_\pi$ in eqs.(15) represents the effect of (pion) loops with intermediate $\Delta(1232)$ states [14] consistently truncated at order q^4 . We should stress that the decomposition of the loop and counter term pieces at order q^4 has, of course, no deeper physical meaning but will serve us to separate the uncertainties stemming from the coefficients accompanying the various contact terms. Notice that from now on we will omit the superscript 'r' on $\delta\bar{\alpha}_i^r(\lambda)$ and $\delta\bar{\beta}_i^r(\lambda)$ appearing in eqs.(15).

The numerical results for the various polarizabilities now depend on the knowledge of the contact terms c_2 , c^+ , $\delta\bar{\alpha}_i(\lambda)$ and $\delta\bar{\beta}_i(\lambda)$ for a given choice of the scale λ . As already stated, ideally the λ -dependence from the loops is cancelled by the one from the corresponding contact terms, as detailed in [16], when one is able to fit all low-energy constants from phenomenology. We are not in that fortunate position but in some cases must resort to the resonance saturation hypothesis [8,9] to estimate some of the constants. In that case, the actual value of a certain low-energy constant is given as a sum of a dominating resonance contribution at the scale λ equal to some resonance scale. This problem will eventually be cured when sufficiently many accurate low-energy data in the baryon sector will be available. Clearly, the contact terms we are dealing with fall into two categories. While c_2 and c^+ only enter via higher order (in $1/m$) vertices in the loop diagrams, $\delta\bar{\alpha}_i(\lambda)$ and $\delta\bar{\beta}_i(\lambda)$ are "genuine" new contact terms of order q^4 . We will therefore discuss the sensitivity on these separately. Note, however, that only the total result of order q^4 is of physical relevance.

The constant c^+ can be related to the isospin-even S-wave pion-nucleon scattering length a^+ [17],

$$c^+ = \frac{256\pi^2 F_\pi^4 a^+ - 3g_A^2 M_\pi^3}{32\pi M_\pi^2 F_\pi^2 (1 - M_\pi/m)} = -0.28 \dots - 0.42 \text{ fm} \quad (19)$$

for $a^+ = -0.83 \pm 0.38 \cdot 10^{-2}/M_\pi$ with $M_\pi = 139.57$ MeV the (charged) pion mass. Inserting the value of c^+ as given from eqs.(19) into eqs.(15,16), one finds that the contributions proportional to c^+ to the electromagnetic polarizabilities are negligible since they are less than $0.15 \cdot 10^{-4} \text{ fm}^3$ in magnitude. If we take only the leading order relation $c^+ = F_\pi^2 a^+ / 8\pi M_\pi^2$, which is allowed to the order we are working, then the c^+ contribution to the electromagnetic polarizabilities is even smaller and only $0.06 \cdot 10^{-4} \text{ fm}^3$ in magnitude. To determine c_2 , we make use of the resonance saturation hypothesis and get contributions from the $\Delta(1232)$ as well as the Roper $N^*(1440)$,

$$\begin{aligned} c_2^\Delta &= \frac{g_A^2 m}{4m_\Delta^2} \left[\frac{m_\Delta + m}{m_\Delta - m} - 4Z^2 \right] \\ c_2^{N^*} &= \frac{Rg_A^2 m}{8(m_{N^*}^2 - m^2)} \end{aligned} \quad (20)$$

where Z parametrizes the off-shell behaviour of the spin-3/2 Rarita-Schwinger field in the $\Delta N\pi$ -vertex and we have used $g_{\pi N\Delta} = 3g_{\pi N}/\sqrt{2}$ as well as $g_{\pi NN^*} = \sqrt{R} g_{\pi N}/2$. Empirically, Z is not known very accurately, $-0.8 \leq Z \leq 0.3$ [18], and R varies from 0.25 to 1.²⁾ Notice that the Roper contribution is more than one order of magnitude smaller than the one from the Δ . If one treats the Δ non-relativistically (isobar model), one arrives at

$$c_2 = \frac{g_A^2}{2(m_\Delta - m)} = 0.59 \text{ fm} \quad (21)$$

for $g_A = 1.328$ by making use of the Goldberger-Treiman relation. Altogether, c_2 varies between 0.4 and 0.6 fm. The resulting polarizabilities depend only weakly on the actual value of c_2 as shown in fig.5. (for $\lambda = 1.232$ GeV). As discussed before, there is some spurious sensitivity on the value of λ as shown in fig.6. The corresponding bands refer to the choice of $g_A = 1.26$ or $g_A = 1.328$ which are equivalent to the order we are working. While the neutron polarizabilities are quite insensitive to the choice of λ , the

²⁾ In ref.[19], a narrower range for Z is given under the assumption of $g_2 = 0$ which seems to be excluded by the data as stressed in [18]. We therefore use the wider range of Z given in [18].

much larger coefficients x_p and x'_p induce some scale dependence for the proton case. Since most of the counterterms are in fact given by Δ exchange, we have chosen in [1] $\lambda = m_\Delta$ (which gives our best values).

The $\Delta(1232)$ enters prominently in the determination of the four low-energy constants from $\mathcal{L}_{\pi N}^{(4)}$. Therefore, we will determine these coefficients at the scale $\lambda = m_\Delta$. In particular, one gets a sizeable contribution to the magnetic polarizabilities due to the strong $N\Delta$ M1 transition. A crude estimate of this has been given in ref.[20] by integrating the M1 part of the total photoproduction cross section for single pion photoproduction over the resonance region,

$$\delta\bar{\beta}_p^\Delta(m_\Delta) = \frac{1}{2\pi^2} \int \frac{d\omega}{\omega^2} \sigma^{M1}(\omega) = 7.0 \cdot 10^{-4} \text{ fm}^3 \quad (22)$$

However, this number is afflicted with a large uncertainty. If one simply uses the Born diagrams with an intermediate point-like Δ , one finds

$$\delta\bar{\beta}_p^\Delta(m_\Delta) = \frac{e^2 g_1^2}{18\pi m^2 m_\Delta^2} \left\{ \frac{m_\Delta^2 + m_\Delta m + m^2}{m_\Delta - m} - 4Y [m_\Delta(2Y + 1) + m(Y + 1)] \right\} \quad (23)$$

with g_1 the strength of the $\gamma N\Delta$ coupling and the off-shell parameter Y is related to the electromagnetic interaction $\mathcal{L}_{\gamma N\Delta}^1$ (see ref.[18] for more details on this). These parameters are not very well determined, a best fit to multipole data for pion photoproduction leads to $3.94 \leq g_1 \leq 5.30$ and $-0.75 \leq Y \leq 1.67$ [18]. The recent PDG tables give $3.5 \leq g_1 \leq 7.5$ [21]. For comparison, SU(4) and chiral soliton models give $g_1 = \kappa^* = 5.0$ which was used e.g. in [10] together with $Y = -1/4$ to estimate the Δ contribution at order q^4 . Inspection of eq.(23) reveals that the large positive values of Y lead to very large negative contributions from the Δ in plain contradiction to the dispersive estimate of eq.(22). If one, however, assumes an universal off-shell parameter for the strong and electromagnetic $N\Delta$ transitions, it is plausible to set $-0.8 \leq Y \leq 0.3$. In that case, $\delta\bar{\beta}^\Delta$ is always positive and varies between 14. and 0.5 as shown in fig.7. A conservative estimate therefore is

$$\delta\bar{\beta}_p^\Delta(m_\Delta) = \delta\bar{\beta}_n^\Delta(m_\Delta) \simeq (7.0 \pm 7.0) \cdot 10^{-4} \text{ fm}^3 \quad (24)$$

invoking isospin symmetry. Clearly, the large range in the value for $\delta\bar{\beta}^\Delta$ is unsatisfactory and induces a major uncertainty in the determination of the corresponding counterterms. In [1], we choose the central value of eq.(24) as our best determination. In the framework of a non-relativistic calculation (isobar model), the corresponding Born term which generates $\delta\bar{\beta}^\Delta$ does not depend on any off-shell parameter and one finds

$e^2 g_1^2 / (18\pi m^2 (m_\Delta - m)) = 12 \cdot 10^{-4} \text{ fm}^3$ (for $g_1 = 5$). In ref.[22], the $\Delta(1232)$ was included in the effective field theory as a dynamical degree of freedom and treated non-relativistically (like the nucleon).³⁾ It has been argued in [22] that the Δ Born graphs have to be calculated at the off-shell point $\omega = 0$. This effect can reduce the large $\delta\bar{\beta}^\Delta$ by almost an order of magnitude. This is reminiscent of the off-shell dependence discussed before. It was already pointed out in ref.[10] that a relativistic treatment of the $\Delta(1232)$ also induces a finite electric polarizability at order q^4 . This contribution depends strongly on the $\gamma\Delta N$ couplings g_1 and g_2 as well as the two off-shell parameters X, Y ,

$$\begin{aligned} \delta\bar{\alpha}^\Delta(m_\Delta) = & \frac{e^2}{18\pi m^2 m_\Delta^2} \left\{ g_1^2 \left[-\frac{m_\Delta^2}{m_\Delta + m} + 4Y(m_\Delta(1 + 2Y) - mY) \right] \right. \\ & + g_1 g_2 \left[\frac{m_\Delta(m - m_\Delta)}{2(m_\Delta + m)} + (X + Y + 4XY)m_\Delta - Y(1 + 2X)m \right] \\ & \left. + \frac{g_2^2}{4} \left[-\frac{4m_\Delta^2 + m_\Delta m + m^2}{4(m_\Delta + m)} + X(1 + 2X)m_\Delta - X(1 + X)m \right] \right\} \end{aligned} \quad (25)$$

The resulting numbers for $\delta\bar{\alpha}^\Delta$ vary between $-6 \cdot 10^{-4} \text{ fm}^3$ and $+4 \cdot 10^{-4} \text{ fm}^3$ for the ranges $-0.8 \leq X, Y \leq 0.4$, $4 \leq g_1 \leq 5$ and $4.5 \leq g_2 \leq 9.5$. In the absence of more stringent bounds on these parameters, we will set $\delta\bar{\alpha}_{p,n}^\Delta(m_\Delta) = 0$ and assign the theoretical predictions for the electric polarizabilities an error of $\pm 2 \cdot 10^{-4} \text{ fm}^3$ accordingly.

Another contribution to the coefficients $\delta\bar{\alpha}_i(\lambda)$ and $\delta\bar{\beta}_i(\lambda)$ comes from loops involving charged kaons [24]. Since we are working in SU(2), the kaons and etas are frozen out and effectively give some finite contact terms. To improve the estimate given in [1], we include the average nucleon- Λ, Σ^0 mass splitting,

$$\Delta = \frac{1}{2}(m_\Lambda + m_{\Sigma^0} - 2m) = 216 \text{ MeV} \quad (26)$$

For the electric and magnetic polarizabilities, this leads to

$$\begin{aligned} \delta\bar{\alpha}_p^K(m_\Delta) &= \frac{C}{6\pi}(D^2 + 3F^2)F_1(M_K, \Delta), & \delta\bar{\alpha}_n^K(m_\Delta) &= \frac{C}{4\pi}(D - F)^2 F_1(M_K, \Delta) \\ \delta\bar{\beta}_p^K(m_\Delta) &= \frac{C}{6\pi}(D^2 + 3F^2)F_2(M_K, \Delta), & \delta\bar{\beta}_n^K(m_\Delta) &= \frac{C}{4\pi}(D - F)^2 F_2(M_K, \Delta) \\ F_1(M_K, \Delta) &= \frac{9\Delta}{\Delta^2 - M_K^2} + \frac{10M_K^2 - \Delta^2}{(M_K^2 - \Delta^2)^{3/2}} \arccos \frac{\Delta}{M_K} \\ F_2(M_K, \Delta) &= \frac{1}{\sqrt{M_K^2 - \Delta^2}} \arccos \frac{\Delta}{M_K} \end{aligned} \quad (27)$$

³⁾ Notice that such an approach does not have a consistent chiral power counting as shown in [23]. It might be justified in the limit of an infinite number of colors where the nucleon and the Δ are degenerate in mass.

If one sets $m_\Lambda = m_{\Sigma^0} = m$ (which is consistent to the order we are working), one recovers eq.(14) of ref.[1]. With $D = 0.8$, $F = 0.5$ and $M_K = 495$ MeV one finds $\delta\bar{\alpha}_p^K(m_\Delta) = 1.31 \cdot 10^{-4} \text{ fm}^3$ and $\delta\bar{\alpha}_n^K(m_\Delta) = 0.13 \cdot 10^{-4} \text{ fm}^3$. The corresponding numbers for the kaon contributions to the magnetic polarizabilities are a factor $F_2/F_1 = 0.12$ smaller. In the case of mass degeneracy for baryons this factor is exactly $1/10$ [24]. The values based on (27) might, however, considerably overestimate the kaon loop contribution. Integrating e.g. the data from ref.[27] for $\gamma p \rightarrow K\Lambda, K\Sigma^0$, one gets a much smaller contribution since the typical cross sections are of the order of a few μbarn . This points towards the importance of a better understanding of SU(3) breaking effects. At present, we can not offer a solution to resolve this discrepancy.⁴⁾

We now are in the position to give a prediction for the electromagnetic polarizabilities. As our central (best) values we take the ones from [1]. The main sources of uncertainty stem from the choice of the value for g_A , the scale λ and the kaon and Δ contributions to the various low-energy constants. Adding these in quadrature leads to

$$\begin{aligned}\bar{\alpha}_p &= (10.5 \pm 2.0) \cdot 10^{-4} \text{ fm}^3 & \bar{\beta}_p &= (3.5 \pm 3.6) \cdot 10^{-4} \text{ fm}^3 \\ \bar{\alpha}_n &= (13.4 \pm 1.5) \cdot 10^{-4} \text{ fm}^3 & \bar{\beta}_n &= (7.8 \pm 3.6) \cdot 10^{-4} \text{ fm}^3\end{aligned}\tag{28}$$

which with the exception of $\bar{\beta}_n$ agree with the empirical data [25]

$$\begin{aligned}\bar{\alpha}_p &= (10.4 \pm 0.6) \cdot 10^{-4} \text{ fm}^3, & \bar{\beta}_p &= (3.8 \mp 0.6) \cdot 10^{-4} \text{ fm}^3 \\ \bar{\alpha}_n &= (12.3 \pm 1.3) \cdot 10^{-4} \text{ fm}^3, & \bar{\beta}_n &= (3.5 \mp 1.3) \cdot 10^{-4} \text{ fm}^3.\end{aligned}\tag{29}$$

making use of the dispersion sum rules [12,26] $(\bar{\alpha} + \bar{\beta})_p = (14.2 \pm 0.3) \cdot 10^{-4} \text{ fm}^3$ and $(\bar{\alpha} + \bar{\beta})_n = (15.8 \pm 0.5) \cdot 10^{-4} \text{ fm}^3$.⁵⁾ Notice that we have added the systematic and statistical errors of the empirical determinations in quadrature. Clearly, an independent determination of the electric and magnetic nucleon polarizabilities would be needed to further tighten the empirical bounds on these fundamental quantities. This was also stressed in ref.[22]. It is worth to point out that the uncertainties given in (28) do not include effects of two (and higher) loops which start out at order q^5 . We do not expect these to alter the prediction for the electric polarizabilities significantly [1]. Such an investigation is underway but goes beyond the scope of this paper. Notice also that at present the theoretical uncertainties are larger than the experimental ones (if one

⁴⁾ We are grateful to Anatoly L'vov for drawing our attention to this problem.

⁵⁾ Notice that the uncertainty on the sum rule value for the neutron is presumably underestimated since one has to use deuteron data to extract the photon-neutron cross section.

imposes the sum rules for $(\bar{\alpha} + \bar{\beta})$). That there is more spread in the empirical numbers when the dispersion sum rules are not used can e.g. be seen in the paper of Federspiel et al. in ref.[25].

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FIGURE CAPTIONS

- Fig.1 Feynman rules for the $1/m$ suppressed propagators and vertices. Solid, dashed and wiggly lines refer to nucleons, pions and photons, in order. Pion momenta are denoted by q_i ($i = 1, 2, 3$), nucleon momenta by ℓ , p_1 , p_2 , and isospin indices by a, b, c .
- Fig.2 The spin-averaged Compton amplitude for the proton normalized to one at $\omega = 0$. (a) $A(z)$ and $A(\zeta)$ compared to the data for $0 \leq \omega \leq 150$ MeV. (b) $A(z)$ and $A(\zeta)$ compared to the quadratic approximation and to the data for $70 \leq \omega \leq 150$ MeV.
- Fig.3 The spin-averaged Compton amplitude for the neutron.
- Fig.4 The real part of $A_p(\omega)$ normalized to one at $\omega = 0$ for $\omega > 140$ MeV in comparison to the data.
- Fig.5 Dependence of the electromagnetic polarizabilities on c_2 for $\delta\bar{\alpha}_i(m_\Delta) = \delta\bar{\beta}_i(m_\Delta) = 0$ ($i = p, n$) in units of 10^{-4} fm^3 .
- Fig.6 Dependence of the electromagnetic polarizabilities on λ for $\delta\bar{\alpha}_i(\lambda) = \delta\bar{\beta}_i(\lambda) = 0$ ($i = p, n$) in units of 10^{-4} fm^3 . The upper/lower rim of the corresponding bands refers to $g_A = 1.328 / 1.26$, in order.
- Fig.7 $\delta\bar{\beta}^\Delta(g_1, Y)$ in units of 10^{-4} fm^3 for $3.8 \leq g_1 \leq 5.3$ and $-0.8 \leq Y \leq 0.3$.

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